Multifragmentation: nuclei break up like percolation clusters

1986 J. Phys. A: Math. Gen. 19 L917

(http://iopscience.iop.org/0305-4470/19/15/010)

J. Phys. A: Math. Gen. 19 (1986) L917–L921. Printed in Great Britain

LETTER TO THE EDITOR

Multifragmentation: nuclei break up like percolation clusters

Xavier Campi

Division de Physique Theorique†, Institut de Physique Nucléaire, Université Paris-Sud, 91406 Orsay Cedex, France

Received 4 July 1986

Abstract. The moments of the cluster size distribution are studied for single events in a finite-size percolation model and in multifragmentation of atomic nuclei. It is shown that both systems break up in roughly the same way.

Atomic nuclei break into lighter nuclei when hit by energetic projectiles. The number and size of nuclear fragments depend on the energy of the collision. When projectile and target interact weakly one observes experimentally one large target residue, plus some very light fragments. When the interaction is strong, there is no large residue but various light and medium size fragments. In very violent collisions, one even sees a complete disintegration of nuclei into free nucleons (Hufner 1985).

The evolution of the fragment's mass distribution recalls qualitatively that of percolation phenomena (Stauffer 1985). In fact, percolation models have already been proposed (Bauer *et al* 1985, Campi and Desbois 1985) to describe fragment mass distributions produced in inclusive experiments. (In inclusive experiments, data from all types of collisions are recorded, regardless of the energy displayed in the collision.) The physical picture that supports that interpretation is the following: before the collision takes place, the nucleons of the target form a single connected cluster. Each nucleon is a site, which is bounded by a few neighbouring nucleons. During the collision, projectile and target interpenetrate and a cascade of nucleon–nucleon collisions develops. Recoil nucleons are ejected out of the nuclear volume and nucleon-nucleon bonds are broken. The degree of damage produced in the target depends on the kinetic energy of the projectile and the overlap of the projectile with the target. Nucleons that remain in a finite nuclear volume form clusters that can be observed as bound nuclei. In practice, the realisation of such ideas in a concrete nuclear percolation model requires a detailed description of the collision dynamics, as well as a careful definition of the model space and linkage conditions. In particular, the relation between the concentration parameter p in percolation and standard measurable quantities in nuclear physics is very delicate to handle.

The relevance of percolation ideas in nuclear fragmentation can be investigated better by examining cross relations between various moments of the fragment size distribution. We show in this letter that experimental data have strong similarities with the predictions of finite-size percolation models.

The standard method used in condensed matter physics to determine the nature of a critical phenomenon in an infinite system is to look at critical exponents. For

† Laboratoire associe au CNRS.

many physical systems it is possible to measure a set of exponents allowing the classification into a definite class of phenomena (liquid–gas phase transition, percolation, etc) (Domb and Green 1972-6). We recall first a few basic concepts and formulae.

The moments of the cluster size distribution

$$M_k(\varepsilon)=\sum s^k n(s,\varepsilon) \tag{1}$$

diverge for $k>1$ at critical points $\varepsilon=0$, with critical exponents

$$M_k \propto \varepsilon^{-\mu_k}. \tag{2}$$

In formula (1), $n(s,\varepsilon)$ is the mean number of clusters of size s at fixed small $\varepsilon=p-p_c$ or T_c-T. The sum runs over all finite-size clusters. We recall, for instance, that

$$M_2 \propto \varepsilon^{-\gamma} \tag{3}$$

where γ is a critical exponent.

In general

$$\mu_k=-(\tau-1-k)/\sigma \tag{4}$$

assuming the scaling property (Stauffer 1985)

$$n(s,\varepsilon)\sim s^{-\tau}f(\varepsilon s^{\sigma}) \tag{5}$$

where τ and σ are two critical exponents. We recall in passing that M_0 (the mean number of clusters) and M_1 (the mean size) do not diverge at critical points.

The critical behaviour of the size of the largest (infinite) cluster defines another exponent β:

$$P(\varepsilon)\propto \varepsilon^{\beta} \tag{6}$$

for $\varepsilon>0$. When $\varepsilon<0$, no infinite cluster exists, but we can still study the size of the typical finite cluster:

$$\tilde{s}(\varepsilon)\propto \varepsilon^{-(\gamma+\beta)}. \tag{7}$$

We discuss now how to handle these ideas in the nucleus break-up problem. Formulae (1)–(7) cannot be directly applied because we ignore how to classify the experimental events according to $p-p_c$ or T_c-T. In addition, we have to keep in mind that nuclei are very small systems, for which the above relations can only be qualitatively fulfilled.

The experimental data we analyse (Waddington and Freier 1985) consist of about 400 collision events, in which gold nuclei (formed by $N=118$ neutrons plus $Z=79$ protons) break up into lighter fragments. Event by event, the size (in fact, the charge Z) of all fragments has been measured. There is no model-independent way to classify the events with respect to p_c or T_c. This means that we cannot define an averaged multiplicity $n(s,\varepsilon)$ for fixed ε. We avoid this difficulty studying the moments' distribution of single events:

$$M_k^j=\sum s^k m^j(s) \tag{8}$$

where $m^j(s)=0,1,2,\ldots,$ is the number of fragments of size s that appear in the event j. Here the sum runs over all fragments, excluding the heaviest one produced in the event. It is more natural to work with normalised moments

$$S_k^j=M_k^j/M_1^j. \tag{9}$$

Of course, in our finite system, the moments remain finite, even for $k>1$. However, if the system keeps some trace of critical behaviour, for some particular events S_k^j

should be much larger than the average. More qualitatively, we can check if the system still behaves approximately like (3) and (4), plotting ln S_k^j against ln $S_{k'}^j$ (for k, $k' > 1$). A linear and strong correlation between the j points would mean a positive answer.

Figure 1 shows the correlation for S_3^j against S_2^j. In (a) the nuclear fragmentation events are shown. In (b), each point is a Monte Carlo simulation of a simple cubic lattice bond percolation model containing about the same number of sites as nucleons in our breaking nucleus ($A = 6^3$). The values of p are randomly distributed between 0 and 1. Events close to the critical region are represented by points with the largest values of S_3 and S_2. Above and below the critical zone, points fall closer to the origin. In nuclei break-up, gentle and violent events fall near the origin. 'Critical' events, corresponding to break up into two or three medium size fragments, again give the largest S_k.

We remark in figure 1 that the points are well correlated, in nuclei as well as in the percolation model. This is quite unexpected for such small systems. In addition, if we believe that formulae (3) and (4) still make sense, the slope $\lambda_{3/2}$ of the straight line is related to the critical exponents by

$$\lambda_{3/2} = 1 + 1/\sigma\gamma. \tag{10}$$

We see that the slope is about the same for the nuclear data as for the percolation simulation: $\lambda_{3/2} = 2.22 \pm 0.1$. This number is to be compared with $\lambda_{3/2} = 2.25$ in infinite percolation models (Stauffer 1985). In contrast, $\lambda_{3/2} = 2.5$ for liquid-gas phase transitions in the mean-field approximation and $\lambda_{3/2} = 3$ for percolation on the Bethe lattice. We recall that most calculations of nuclear fragmentation are performed in a mean-field approximation (Goodman *et al* 1984).

We also remark in figure 1 on a slight change in the slope of the straight line defined by the most 'critical' events. This is visible in both nuclear and percolation events, although more marked in the former. We believe that this is a manifestation of the finite size of the system, but this calls for closer investigation.

The validity of the general expression (4), which follows from the scaling hypothesis (5), can be checked comparing higher moments. This is done in figure 2 for S_5 plotted

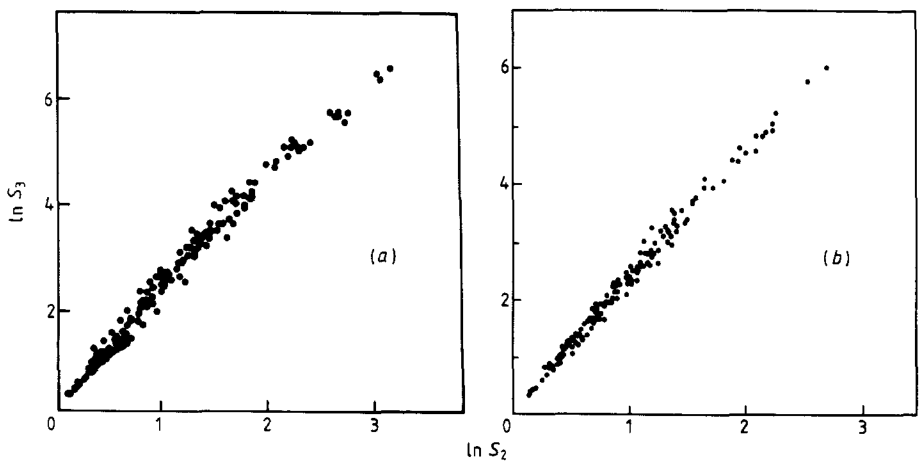

Figure 1. Single event moments S_3^j plotted against S_2^j for nucleus break up (a) and for a Monte Carlo simulation in a cubic bond percolation model containing 216 sites and randomly distributed values of $0 < p < 1$ (b).

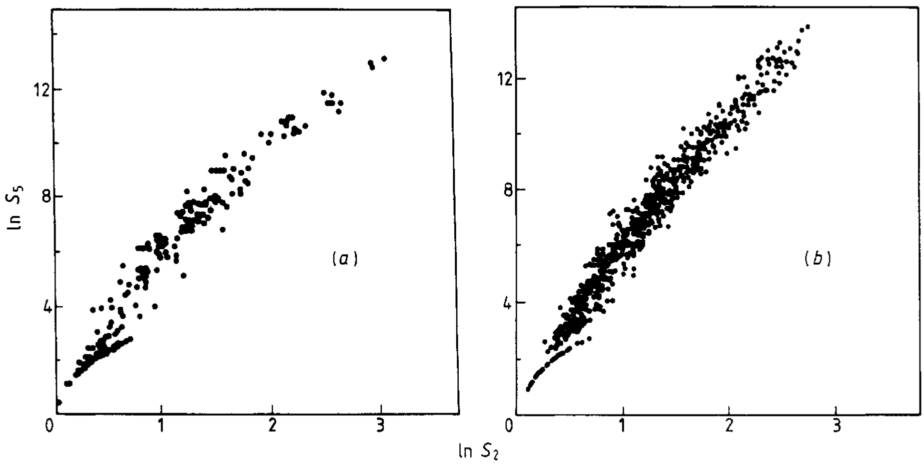

Figure 2. Same as figure 1 for S_5^j and S_2^j. See text for the curious correlation near the origin.

against S_2. We see that in both systems the slope $\lambda_{5/2} = 4.6 \pm 0.2$, in agreement with the prediction 4.75 of (4) taking $\tau = 2.2$, the standard value in three-dimensional percolation. The value predicted in the mean-field approximation for a liquid–gas phase transition is $\lambda_{5/2} = 5.5$ taking $\tau = 7/3$. The intriguing points that draw an arc of a circle close to the origin are events with only $s = 1$ and $s = 2$ clusters contributing to S_k, i.e. very gentle ($p \simeq 1$) and very violent ($p \simeq 0$) ones.

Finally we study the 'critical' behaviour (6) and (7) of the 'infinite' and 'typical' clusters which we identify in our finite-size systems with the largest cluster produced in the event. Plotting the size of that cluster against S_2 we get a much weaker correlation than before. This lack of correlation is due to the large fluctuations of P and $\tilde{S}$ in our small systems. A remedy is to average over events of the same type, e.g. averaging over events with the same S_1. (We have checked in the percolation model that S_1 is a regular decreasing function of p, particularly for $p > p_c$.) The result is shown in figure 3, where the points represent average values of two or more events. We remark again that the nucleus and percolation models give about the same correlations. One clearly distinguishes two branches. The upper one concerns events with $p > p_c$. From (3) and (6) the predicted slope is $-\beta/\gamma$, i.e. too small to be measured accurately. The lower branch represents events below p_c for which the size of the largest cluster may be governed by (7). The observed slope $\lambda = 1.2 + 0.1$ has the predicted value in percolation $\lambda = 1 + \beta/\gamma = 1.2$ ($\lambda = 1.5$ in the mean-field approximation).

In summary, the moments of cluster size distributions in individual events are strongly correlated. This is true even in very small systems containing only a few hundred particles or sites. Correlations appear to be about the same in a heavy nucleus break up as in a finite-size standard percolation model. When these correlations are interpreted as remanents of critical phenomena, the deduced values of critical exponents are very similar and close to those of percolation for infinite systems, and different from the mean-field approximation.

The use of single event moments seems to be a powerful method for finding critical exponents when data cannot be ordered according to $p - p_c$ or to $T_c - T$. When applied to nuclear physics, this method provides a model-independent way of analysing the complicated data coming from the new exclusive multifragmentation experiments.

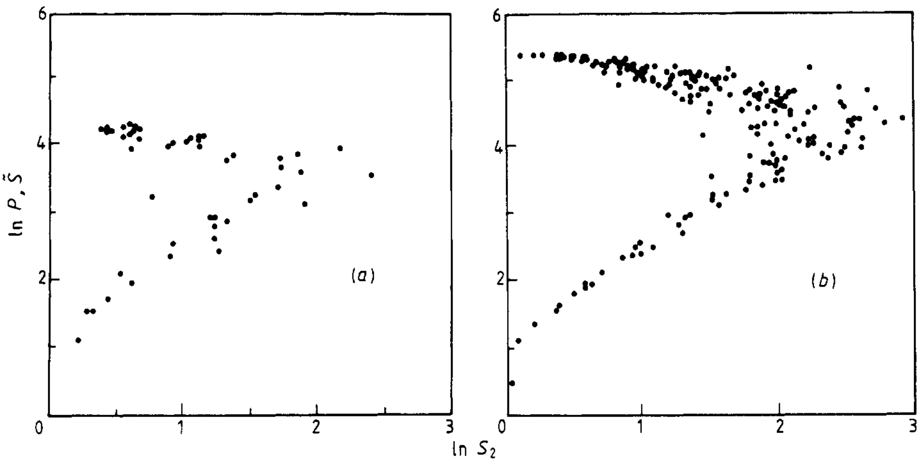

Figure 3. The size of the largest cluster produced per event as a function of S_2. Each point represents the average over events with same S_1. (a) Largest nuclear fragment charge. The average is over 376 events. (b) Largest cluster size in a cubic bond percolation model containing 216 sites and for randomly distributed values of $0<p<1$. The average is over 4000 events. Only the slopes of the curves can be compared because of different system sizes.

The author is indebted to M T Commault for help in the Monte Carlo computer simulations. He would also like to thank B Derrida, H Herrmann and H Hufner for discussions and C J Waddington for communicating the experimental data. Special thanks go to D Stauffer for encouragement and a critical reading of the manuscript.

References

Bauer W, Dean D R, Mosel U and Post U 1985 *Phys. Lett.* **150B** 53-6

Campi X and Desbois J 1985a *Proc. 23rd Bormio Conf.* (Milano: Ric. Sci. Educ. Perm.) pp 497-511

—— 1985b *Phase Space Approach to Nuclear Dynamics* (Singapore: World Scientific) pp 238-50

Domb C and Green M S (ed) 1972-6 *Phase Transitions and Critical Phenomena* vols 1-6 (New York: Academic)

Goodman A L, Kapusta J I and Mekjian A Z 1984 *Phys. Rev.* C **30** 851-65

Hufner H 1985 *Phys. Rep.* **125** 129-85

Stauffer D 1985 *Introduction to Percolation Theory* (London: Taylor and Francis)

Waddington C J and Freier P S 1985 *Phys. Rev.* C **31** 888-95